Operator Theory and Mathematical Physics

International conference honoring Pavel Exner's contributions in mathematical physics on the occasion of his 80th birthday

Prague, May 24-29, 2026

BOOK OF ABSTRACTS

Invited talks

Spectral statistics of quantum graphs with preferred orientation

Ram Band

Technion – Israel Institute of Technology, Haifa, Israel

Quantum graphs with vertex conditions of preferred orientation have been introduced by Exner and Tater a few years ago. Since then they are an interesting topic of study both theoretically and from the applied perspective. We are interested in studying the spectral statistics of such graphs from the viewpoint of the Bohigas-Giannoni-Schmit conjecture. Motivated by this conjecture, we compare the spectral statistics of preferred orientation graphs with the corresponding RMT ensembles, and along this way discover two surprises.

This is a joint work with Pavel Exner, Divya Goel and Aviya Strauss.

Spectral theory for Dirac operators with singular potentials

Jussi Behrndt

Graz University of Technology, Austria

In this talk, we discuss qualitative spectral properties of self-adjoint Dirac operators. We first briefly review some of the standard results for regular potentials and turn to more recent developments afterwards. Our main objective in this lecture is to discuss Dirac operators with singular potentials supported on curves or hyperplanes, where it is necessary to distinguish the so-called non-critical and critical cases for the strength of the singular perturbation. In particular, it turns out that Dirac operators with singular potentials in the critical case have some unexpected spectral properties that depend on the geometry of the support of the perturbation. This talk is based on joint works with B. Benhellal, P. Exner, M. Holzmann, V. Lotoreichik, T. Ourmieres-Bonafos, and K. Pankrashkin.

Breaking Boundaries: Closed Nodal Lines on Doubly-Connected Domains

Pedro Freitas

University of Lisbon, Portugal

In 1967, L.E. Payne conjectured that the nodal line of any second eigenfunction of the Dirichlet Laplacian on a planar domain must touch the boundary at exactly two points. While this holds for convex domains (Melas 1992), it was shown to fail for certain multiply-connected domains by M. Hoffmann-Ostenhof, T. Hoffmann-Ostenhof, and N. Nadirashvili in 1997. A central question raised in their work was the minimal connectivity required for such a counterexample to exist.

Until recently, the best-known result required four holes (Dahne, Gómez-Serrano, and Hou, 2021). In this talk, we introduce a new family of counterexamples showing that one hole is enough for the conjecture to fail. This effectively bridges the gap between known counterexamples and the still-open simply-connected case.

Furthermore, we revisit Payne's parallel assertion for the Neumann problem which claimed that a second closed nodal line cannot exist for general planar domains. We identify a subtle gap in the original reasoning and, inspired by our Dirichlet construction, present a doubly-connected domain that provides a definitive counterexample for the Neumann case.

This is based on joint work with Roméo Leylekian (Lisbon).

Essential self-adjointness for a class of pseudo-differential operators perturbed by strongly singular potential coefficients

Fritz Gesztesy

Baylor University, Waco, TX, USA

For $\emptyset \neq X \subset \mathbb{R}^n$ discrete, $n \in \mathbb{N}$, we characterize the distributions in $\mathcal{D}'(\mathbb{R}^n)$ with support contained in X and subsequently use this result to prove that $C_0^\infty(\mathbb{R}^n \backslash X)$ is dense in the fractional Sobolev space $H^s(\mathbb{R}^n)$ if and only if $s \in (-\infty, n/2]$. This fact is then used to show that

$$(-\Delta)^s\big|_{C_0^\infty(\mathbb{R}^n \backslash X)} \text{ is essentially self-adjoint in } L^2(\mathbb{R}^n)$$
$$\text{if and only if } s \in (0, n/4] \text{ (i.e., if and only if } n \geq 4s),$$

and analogously for the operator $\big(-\Delta + m^2 I\big)^s\big|_{C_0^\infty(\mathbb{R}^n \backslash X)}$, $m \in (0, \infty)$.

In addition, for $n \in \mathbb{N}$, $s \in (0, n/4)$, applying the fractional Birman–Hardy–Rellich-type inequality combined with Kato–Rellich–Wüst perturbation theory, we prove essential self-adjointness (resp., self-adjointness) of

$$\big[(-\Delta)^s + c|\,\cdot\,|^{-2s}\big]\big|_{H^{2s}(\mathbb{R}^n)}$$

in $L^2(\mathbb{R}^n)$ if $c \in [-C_{n,s}, C_{n,s}]$ (resp., $c \in (-C_{n,s}, C_{n,s})$). Here $C_{n,s}$ is given by

$$C_{n,s} = 2^{2s}\Gamma((n/4) + s)/\Gamma((n/4) - s), \quad s \in (0, n/4).$$

As an introduction into this subject we will also discuss the case of even-order, strongly singular ODE operators on the half-line $(0, \infty)$.

This talk is based on various joint work with Markus Hunziker, Dorina Mitrea (Baylor Univ., TX, USA) and Gerald Teschl (Univ. of Vienna, Austria).

Extremal spectral problems for quantum graphs

Evans Harrell

Georgia Institute of Technology, Atlanta, GA, USA

"Spectral optimization" refers to the study of how shapes affect eigenvalues of self-adjoint operators by trying to maximize or minimize an eigenvalue, or some combination of eigenvalues, under reasonable constraints. This is a well-developed subject for the Laplacian on domains and manifolds, and some other PDEs and ODEs, but only in recent years have such questions been posed on metric graphs. Remarkably, the answers are sometimes quite different from the classical cases.

I'll review some known optimal spectral estimates for metric graphs and will then discuss recent work on optimal ratios of eigenvalues and gaps between eigenvalues. For example, I will prove that the Dirichlet metric trees that produce the largest ratio of the first two eigenvalues are equilateral stars, and will pursue Weyl-sharp estimates for arbitrary eigenvalue ratios. If time permits I will also describe some optimizers for ratios and gaps for quantum graphs that include potential energies.

On weighted Kirchhoff Laplacians on metric graphs

Aleksey Kostenko

University of Ljubljana, Slovenia

In this talk, I will review some of my joint work with Pavel Exner, Mark Malamud, and Noema Nicolussi. The boundary triplets approach turns out to be very efficient when dealing with Laplacians on metric graphs, as it establishes strong connections with weighted Laplacians on (discrete) graphs. It turns out that the underlying graphs, when viewed as metric spaces, are quasi-isometric, which indicates further connections between the corresponding random walks (on discrete graphs) and Brownian motion (on metric graphs).

The main focus of this talk will be on the stability of certain properties (recurrence, stochastic completeness, etc.) and their implications for spectral properties.

Szegö limit theorem and Heisenberg Laplacian

Ari Laptev

Imperial College London, UK

The aim of this paper is to obtain a version of the classical Szegö limit theorem, where instead of the operator of second derivative on a circle we consider the Heisenberg-Hörmander Laplacian in $L^2(\mathbb{R}^3)$. Besides, we derive a sharp inequality for convex functions that we call Szegö inequality.

SPECTRAL ASYMPTOTICS FOR STRONGLY COUPLED INTERACTIONS

Konstantin Pankrashkin

Carl von Ossietzky University of Oldenburg, Germany

I review classical and recent results on eigenvalue asymptotics in the strong-coupling regime for "zero-range-type" operators including delta-interactions, Robin Laplacians, and Dirac operators. Although these classes of problems now appear quite different, they share a common origin in work initiated during my visit to Pavel Exner in May 2012.

THE NEVER-ENDING DIRICHLET STORY

Olaf Post

University of Trier, Germany

In this talk, I will comment on a current project with Pavel Exner that we work on already since a long time. When I started collaborating with Pavel (Pavel was as old as I am now) we first considered branched quantum wave guides (graph-like spaces, thick graphs — however you call it) with Neumann boundary conditions shrinking to the underlying metric graph skeleton. Later, Pavel and I discussed the Dirichlet case which is far more sensitive to the geometry of the branched wave guides. I will present known results and what is still open in this problem.

ISOSCATTERING QUANTUM GRAPHS AND MICROWAVE NETWORKS – ARE THERE ANY ISOSCATTERING GRAPHS THAT ARE NON-ISOSPECTRAL?

Leszek Sirko

Institute of Physics, Polish Academy of Sciences, Warsaw, Poland

We present a new approach to constructing general families of isoscattering graphs. Our concept of germ graphs and the Titchmarsh–Weyl M-function provides a powerful tool for exploring graph theory. We demonstrate how this formalism can be expanded to encompass dissipation, thereby opening up new avenues for the understanding and prediction of the behaviour of complex networks. To validate our theoretical predictions, we have developed microwave networks that emulate open quantum graphs with dissipation. We also use the M-function formalism to study various realisations of isoscattering and non-isospectral quantum graphs. The analysed graphs possessed the same total length but different topologies.

This is a joint work with Pavel Kurasov (Stockholm University), Omer Farooq (Institute of Physics, Polish Academy of Sciences), Szymon Bauch (Institute of Physics, Polish Academy of Sciences), Mats-Erik Pistol (Solid State Physics, Lund University), Matthew de Courcy-Ireland (Stockholm University and Nordic Institute for Theoretical Physics, Stockholm) and Michał Ławniczak (Institute of Physics, Polish Academy of Sciences).

Contributed talks

Coherent frames with zero Beurling density

Ingrid Beltita

Institute of Mathematics "Simion Stoilow" of the Romanian Academy, Bucharest, Romania

We show the existence of a coherent frame in the orbit of a 1-connected unimodular solvable Lie group of exponential growth, given by a square -integrable representation in the weak sense, and for which the lower Beurling density of its index set is zero.

Planarity Criteria for Metric Graphs

Alice Brolin

Stockholm University, Sweden

The Colin de Verdière parameter is a number assigned to discrete graphs which equals the maximal multiplicity of the second eigenvalue of a certain family of Laplacian matrices related to the graph. We show that the Colin de Verdière parameter can be obtained in the setting of metric graphs by looking at the maximal multiplicity of the second eigenvalue for Laplacians on metric graphs with delta couplings at the vertices. Two different families of Laplacians, as well as a family of Schrödinger operators, all leading to the Colin de Verdière parameter are presented. This talk is based on joint work with Pavel Kurasov.

Homogenization regime for extended Aharonov-Bohm fluxes

Domenico Cafiero

Polytechnic University of Milan, Italy

We consider a non-relativistic quantum particle in $\mathbb{R}^2$ in the presence of several regularized Aharonov-Bohm fluxes. We investigate the so-called homogenization limit within a scaling regime where the intensities of the single fluxes, the support of the regularization and the distances between the centres go to zero as the number of fluxes grows, while the total flux remains finite. We show that the Hamiltonian converges (in the sense of Gamma-convergence of quadratic forms) to a Schrödinger operator with a regular "mean-field" magnetic potential. Consequently, we obtain the strong resolvent convergence of the corresponding operators. This represent a first step towards the analysis of many singular AB fluxes.

Based on a joint work with Michele Correggi and Davide Fermi (Polytechnic University of Milan).

The Dark Side of Quantum Hybrids: From Linear Spectral Theory to Nonlinear Ground States on Thin Structures

Raffaele Carlone

University of Naples Federico II, Italy

Quantum hybrid spaces — systems obtained by coupling lower-dimensional structures such as half-lines to planar domains through point interactions — have attracted considerable attention as idealized models of quantum wires, junctions, and nanostructures. A rigorous mathematical framework for the linear theory of such systems is provided by the boundary triple formalism, Weyl functions, and Kreĭn-type resolvent formulas, which yield a complete spectral characterization of the self-adjoint Hamiltonians governing the coupled dynamics. This spectral machinery, while mathematically rich, constitutes what one might call the *dark side* of the theory: an essential but largely invisible substrate upon which physically relevant phenomena are built.

In this talk, we move beyond the linear framework and into the light, addressing the nonlinear Schrödinger equation (NLS) with concentrated nonlinearities on hybrid structures. We discuss well-posedness, the existence and characterization of ground states, and the role of the topology and geometry of the underlying domain — including thin deformations and singular limits — in shaping the nonlinear dynamics. The interplay between the Kreĭn resolvent of the linear part and the energy functional of the NLS reveals how spectral thresholds, resonances, and boundary data inherited from the dark side control the stability and blow-up landscape of the nonlinear problem.

This is a joint work with R. Adami, F. Boni, L. Tentarelli, S. Niro, A. Rizzello, and S. Di Giorgio.

Sharp decay and unique continuation at infinity for Dirac operators

Biagio Cassano

University of Campania Luigi Vanvitelli, Caserta, Italy

We determine the asymptotic behaviour at infinity of solutions to perturbed Dirac equations. For the massless Dirac equation with a (possibly non-Hermitian) matrix-valued potential, we determine the maximal exponential decay rate of non-trivial solutions in terms of the decay of the potential, and prove the sharpness of our results in dimensions two and three by constructing explicit examples. These results can be interpreted in terms of unique continuation from infinity and are related to the Landis' conjecture for the Dirac operator. We also establish strong unique continuation results from infinity for Dirac equations with Coulomb-type perturbations, proving the optimality of the critical constant for the validity of the result by means of an explicit example in dimension two.

Opinion forming in a quantum network model

Leonid Chaichenets

Czech Technical University in Prague, Czech Republic

Based on the findings on the dynamics defined by the fully connected graph reported previously, we generalize the model to a class of bi-partition of the initial graph and show a number of effects influencing the dynamics. In particular, we show how the dynamics is influenced by the topology of the two constituting graphs and the relative coupling strength. The scaling properties of the trapping mechanics with respect to the size of the graph and the relative strength of the local couplings are discussed. The implications for the transfer of opinion between likely-minded groups are discussed.

Dirac particle in a time-dependent spherical box

Jaroslav Dittrich

Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czech Republic

Dirac particle confined in a spherical box with a time-dependent radius is considered. Physically observable characteristics as the average kinetic energy, average quantum force acting on the particle by the moving wall, and the trembling motion are analyzed as functions of time. The final results are obtained mostly numerically.

Common work with K. Matchonov and D. Matrasulov (Tashkent).

Robustness of quantum symmetries against perturbations

Paolo Facchi

University of Bari & INFN Bari, Italy

We investigate quantum symmetries in terms of their large-time stability with respect to perturbations of the Hamiltonian. We find a complete algebraic characterization of the set of symmetries robust against a single perturbation and we use such result to characterize their stability with respect to arbitrary sets of perturbations.

Homogenization of point interactions

Davide Fermi

Polytechnic University of Milan, Italy

We discuss the effective behaviour of a non-relativistic quantum particle interacting with a regular electromagnetic field and a large collection of singular zero-range potentials concentrated at discrete points. We focus on a scaling regime in which the number of points increases, while the associated intensities and mutual distances simultaneously vanish, so that the total interaction strength remains finite. Assuming the singular potentials to have negative scattering lengths and to be uniformly distributed, we prove that the associated quadratic forms Γ-converge to a limit comprising an additional electric potential. As a consequence, we establish strong resolvent convergence of the corresponding Hamiltonian operators. In presence of an external trapping potential, this convergence can be strengthened to norm resolvent convergence.

Based on joint work with Domenico Cafiero and Michele Correggi (Politecnico di Milano), arXiv:2603.21400 [math-ph].

Essentially singular limits of Jacobi operators and applications to higher-order squeezing

Felix Fischer

University of Erlangen–Nuremberg, Germany

We study a family of Jacobi operators in which the diagonal entries are multiplied by a coupling parameter $\lambda \geq 0$. Under suitable conditions, the operator is self-adjoint for every $\lambda > 0$, while the formal limit at $\lambda = 0$ is a symmetric Jacobi operator admitting a one-parameter family of self-adjoint extensions. A central ingredient of our analysis is the derivation of uniform bounds for square-summable generalized eigenvectors in the small-λ regime, which combines discrete WKB methods with Airy-function asymptotics. Using these estimates, we analyze the limiting behavior $\lambda \to 0$ in the strong resolvent sense, proving that for every sequence $\lambda_j \to 0$ one can extract a subsequence along which the corresponding Jacobi operators converge to some self-adjoint extension of the limiting operator; conversely, every such extension can be obtained in this way. We call this behavior an essentially singular limit, by analogy with essential singularities in complex analysis. As an application, we study higher-order squeezing operators arising in quantum optics. Using the connection with Jacobi operators, we show that when the relative strength of the free-field term tends to zero, different self-adjoint extensions of the squeezing operator are selected along different sequences. In particular, this limit does not single out a physically distinguished self-adjoint extension, but instead identifies a distinguished subclass of extensions compatible with the underlying symmetry.

Joint work with Daniel Burgarth and Davide Lonigro.

Hall conductance in frustration free models

Martin Fraas

UC Davis, CA, USA

I will describe a new proof that translation-invariant free Fermion frustration-free models have zero Hall conductance. The proof uses the bulk-edge correspondence. The central argument is that in frustration-free models, edge modes can't merge into the ground state band. I will also give an example of a gapped frustration-free model that is nonetheless gapless on the half-plane. This is based on joint work with S. Bachmann and S. Du.

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A Consistent Numerical Approach to Quantum Graph Computations

Roy Goodman

New Jersey Institute of Technology, Newark, NJ, USA

A quantum graph is a linear Schrodinger operator defined on a metric graph. We take the view that any PDE whose spatial derivatives are Laplacian can be formulated on a quantum graph, and that any mathematical question about such problems should be amenable to numerical computation. The QGLAB project set out to make setting up and solving such problems simple, allowing users to easily construct and discretize quantum graphs and their Laplacian operators, solve linear and nonlinear problems, and visualize the results while working at a high level of abstraction. The principal idea behind this package is the rectangular differentiation matrix, with more columns than rows, which allows the vertex conditions to be handled flexibly across a wide variety of problems.

Geometrically induced bound states for Laplacians with oblique transmission conditions

Markus Holzmann

Graz University of Technology, Austria

First, the Laplacian with oblique transmission conditions on an unbounded smooth curve Γ in $\mathbb{R}^2$, which is a compact perturbation of a broken line, is introduced as a self-adjoint operator in $L^2(\mathbb{R}^2)$. While for positive parameters the operator is non-negative, the operator is unbounded from below for negative parameters, and in the latter case the essential spectrum consists of two unbounded intervals that do not overlap. Using the seminal construction by Exner and Ichinose it is shown for negative coupling constants that there is always a geometrically induced eigenvalue in the gap of the essential spectrum, if Γ is not the straight line.

Semiclassical and quantum bounds for Schrödinger operators: From Laplacian, to Fourier multipliers, and general kinetic energies

Dirk Hundertmark

Karlsruhe Institute of Technology, Germany

The Lieb-Thirring (LT) and Cwikel-Lieb-Rozenblum (CLR) bounds for Schrödinger operators are well-known. They bound the sum of the (negative) eigenvalues by a semi-classical phase space integral (LT), respectively the number of negative eigenvalues by the volume of the phase space where the classical energy is negative (CLR). However, it is well-known that the usual CLR bound fails if the dimension is one or two, whereas the semiclassical phase-space volume can still be finite. So what distinguishes low dimensions from higher? Is there a version of both bounds for general kinetic energies? In the CLR case, how can this bound know when it cannot hold, as, e.g., in dimension one and two for the usual Schrödinger operator with the Laplacian as kinetic energy. It turns out that the usual CLR bound is of semiclassical type by mere accident! There is a precise version of the CLR bound which works for much more general energies and it detects precisely when it can, or cannot, hold. It is not a semiclassical bound. Moreover, one can easily read off from the kinetic energy, when this bound holds and when it cannot. Similarly, there is a version of the LT bound which is always of semiclassical type, even for very general kinetic energies.

Asymptotic Properties of Atomic Hamiltonians

Michal Jex

Czech Technical University in Prague, Czech Republic

We present sharp upper bounds for the behaviour of the ground state eigenfunctions of N-electron atoms with general charge Z. This result is generalisation of the result recently accepted for publication about Helium atom (Hundermark-Jex-Lange 2026). The proof relies on careful energy estimates and use of non-conical regions. Furthermore we show why using conical regions can not lead to sharp bounds.

Uneven Magnetic Scaling and High-Flux Spectral Behavior

Ayman Kachmar

American University of Beirut, Lebanon

We study the lowest eigenvalue of the Neumann magnetic Laplacian in a planar domain divided into two regions, with piecewise constant magnetic fields that may scale differently in the inner and outer parts. Our aim is to describe the high-flux limit and determine when the ground-state energy is eventually monotone and when it continues to oscillate. When the outer field is fixed, the lowest eigenvalue exhibits persistent oscillations and the low-energy states localize in the outer region. When the outer field grows more slowly than the inner field, the behavior depends strongly on the geometry: it is eventually monotone for non-circular domains, while oscillations may persist for disks. When the outer field dominates, the problem reduces asymptotically to an effective operator on the inner region. Joint work with E. L. Giacomelli (Milan) and M. P. Sundqvist (Lund).

New Amplitudes Methods for Effective Field Theories

Karol Kampf

Charles University, Prague, Czech Republic

Modern amplitudes methods in QFT will be briefly introduced. These new techniques will then be applied in effective field theories, namely the non-linear sigma model. Its generalization is now known as the soft bootstrap.

Homogenization and operator estimates for Steklov problems in perforated domains

Andrii Khrabustovskyi

University of Hradec Králové, Czech Republic

Let the set Ω_ε be obtained from a domain Ω by removing a family of ε-periodically distributed identical balls. In the perforated domain Ω_ε, one considers the Steklov spectral problem. It is known from [Girouard-Henrot-Lagacé, ARMA (2021)] that, when the holes shrink at a critical rate (so that the surface area of each hole is comparable to the volume of a periodicity cell) the spectrum converges, as $\varepsilon \to 0$, to that of a weighted Laplacian.

In this talk, we present an extension of this result. Under fairly general assumptions on the geometry and distribution of the holes, we prove convergence of the associated resolvent operators in the operator norm. In addition, we obtain quantitative estimates on the Hausdorff distance between the spectra. Notably, our analysis does not require the underlying domain Ω to be bounded.

This is joint work with Jari Taskinen (University of Helsinki).

Soft quantum waveguides and layers

Jan Kříž

University of Hradec Králové, Czech Republic

We consider the Schrödinger operator with confining potentials depending on the distance to the planar curve or spatial surface. For special geometries, we localize the essential spectrum and establish the existence of discrete eigenvalues. This is a joint work with David Krejčiřík and Sylwia Kondej.

Mathematical gems: 40 years of adventures

Pavel Kurasov

Stockholm University, Sweden

The author will describe numerous interactions with Pavel Exner during more than 40 years of mathematical adventures. The focus will be on mathematical gems, which grace lives of mathematicians.

As an example of such germ we shall present figure-eight graph with a family of vertex conditions leading not only to topology change, but also to non-trivial topological Berry's phase. Connection between topology change and non-triviality of the topological phase will be discussed. This is joint work with Vladislav Shubin and Axel Tibbling.

The spectral determinant for second-order elliptic operators on the real line

Jiří Lipovský

University of Hradec Králové, Czech Republic

The spectral determinant for an operator with infinitely many eigenvalues is a generalization of the notion of the determinant of a square matrix. First, we introduce the definition of the spectral determinant. Then we remind a classic result by Levit and Smilansky about the determinant of a Schrödinger operator on an interval. With the methods inspired by this result, we show a formula (and outline its proof) for the determinant of a compact perturbation of a Schrödinger operator on the real line that uses the determinant of a non-perturbed operator and certain Wronskians. Finally, we apply it to a harmonic and anharmonic oscillator. The talk will be based on the joint paper with P. Freitas:

P. Freitas, J. Lipovský, The spectral determinant for second-order elliptic operators on the real line, Lett. Math. Phys. 114 (2024), 65. [arXiv:2405.03469]

Convergence of Schrödinger operators on domains with scaled resonant potentials

Vladimir Lotoreichik

Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czech Republic

We will consider Schrödinger operators on a bounded, smooth domain of dimension $d \geq 2$ with Dirichlet boundary conditions and a properly scaled potential, which depends only on the distance to the boundary of the domain. We will discuss the convergence of these operators as the scaling parameter tends to zero. It turns out that if the scaled potential is resonant, the limit in strong resolvent sense is a Robin Laplacian with boundary coefficient expressed in terms of the mean curvature of the boundary. Remarkably, a counterexample shows that norm resolvent convergence cannot hold in general in this setting. On the contrary, if the scaled potential is non-resonant and its negative part satisfies an explicit smallness assumption, the limit in strong resolvent sense is the Dirichlet Laplacian. We conjecture that we can drop this smallness assumption in the non-resonant case. Our results are largely inspired by the contributions of Pavel Exner and his collaborators on this topic. This talk is based on a joint work with Olaf Post.

On spectral theory of non-selfadjoint 2×2 Dirac-type operators on a finite interval

Anton Lyunov

Meta Platforms, Inc, US

In this talk we investigate the spectral properties of the boundary value problems (BVP) associated with the following 2×2 Dirac type equation:

$$y' + Q(x)y = i\lambda B(x)y, \qquad y = \mathrm{col}(y_1, y_2), \qquad x \in [0, \ell],$$

on a finite interval $[0, \ell]$ subject to the general two-point boundary conditions

$$Cy(0) + Dy(\ell) = 0 \quad \text{with} \quad C, D \in \mathbb{C}^{2\times 2}.$$

Here $Q(\cdot) = (Q_{jk}(\cdot))_{j,k=1}^2$ is an integrable potential matrix and $B(\cdot) = \mathrm{diag}(\beta_1, \beta_2) = B^*(\cdot)$ is a diagonal integrable matrix-function "weight". If $B(\cdot) = \mathrm{diag}(-1, 1)$, this equation turns into 2×2 Dirac equation.

We provide a thorough spectral theory for such operators including asymptotic behavior of the eigenvalues, completeness, and Riesz basis property of the system of root vectors. We also discuss sufficient conditions of the completeness property in the case of non-regular boundary conditions. In this case the completeness property depends also on the potential matrix.

This talk is based on a joint papers with Mark Malamud: [1], [2] and their development.

References

[1] A.A. Lunyov and M.M. Malamud, On the formula for characteristic determinants of boundary value problems for $n \times n$ Dirac type systems and its applications, *Advances in Math.*, **478** (2025), 110389, 96p.

[2] A.A. Lunyov and M.M. Malamud, On the completeness property of root vector systems for 2×2 Dirac type operators with non-regular boundary conditions, *J. Math. Anal. Appl.* **543** (2025), no. 2, part 1, Paper No. 128949, 42p.

TO THE BIRMAN-KREIN-VISIC THEORY

Mark Malamud

St. Petersburg University

Let A be a closed non-negative symmetric densely defined operator in a Hilbert space $\mathfrak{H}$ and let $\mathfrak{H}_1 := \mathrm{ran}(A + I)$. By the Stone – Friedrichs theorem the set $\mathrm{Ext}_A(0, \infty)$ of all nonnegative selfadjoint extensions $\widetilde{A} = \widetilde{A}^*$ of A is nonempty. M. Krein established that the set $\mathrm{Ext}_A(0, \infty)$ forms an operator segment with two endpoints: the maximal (the Friedrichs) and the minimal (the Krein) extensions $\widehat{A}_F$ and $\widehat{A}_K$. They are uniquely characterized by means of the inequalities: $\widehat{A}_K \le \widetilde{A} \le \widehat{A}_F$, $\widetilde{A} \in \mathrm{Ext}_A(0, \infty)$, which are understood in the form sense.

Krein's theory has substantially been completed by M. Vicik and M. Birman.

If A is positive definite, then $\widehat{A}_K$ admits a representation $\widehat{A}_K = \widehat{A}'_K \oplus (\mathbb{O} \upharpoonright \mathfrak{N}_0)$ where $\mathfrak{N}_0 := \ker A^*$. The operator $\widehat{A}'_K$ is called the reduced Krein extension.

M.Krein proved the implication $(I_{\mathfrak{H}} + \widehat{A}_F)^{-1} \in \mathfrak{S}_\infty \implies (I_{\mathfrak{M}_0} + \widehat{A}'_K)^{-1} \in \mathfrak{S}_\infty$. In [1] this result was improved and completed by showing that replacing $\widehat{A}_F$ by A turns the above implication into the equivalence:

$$P_1(I_{\mathfrak{H}} + A)^{-1} \in \mathfrak{S}(\mathfrak{H}_1) \iff (I_{\mathfrak{M}_0} + \widehat{A}'_K)^{-1} \in \mathfrak{S}(\mathfrak{M}_0), \tag{1}$$

where P_1 is the orthoprojection in $\mathfrak{H}$ onto $\mathfrak{H}_1$, $\mathfrak{M}_0 = \mathfrak{N}_0^\perp$, and $\mathfrak{S}$ is any symmetrically normed ideal (including ideals $\mathfrak{S}_p, \Sigma_p, \Sigma_p^0$, etc.).

In accordance with the Grubb result an extension $A_B = A_B^*$ is semibounded below only simultaneously with its boundary operator B (LSB-property of A) whenever $(I_{\mathfrak{H}} + A)^{-1} \in \mathfrak{S}_\infty$.

We will discuss the following themes:

(i) Additional conditions on A that ensure the following equivalence

$$\lambda_n\big(\widehat{A}_F)\big) = a^{-1} n^{1/p} \left(1 + o(1)\right) \iff \lambda_n\big(\widehat{A}'_K\big) = a^{-1} n^{1/p} \left(1 + o(1)\right).$$

which completes (1) in the case of $\mathfrak{S} = \Sigma_p$

(ii) Solution to the abstract Alonso-Simon problem.

(iii) Improvement of certain results by Birman and Grubb regarding the LSB-property of A and its application to elliptic BV problems in unbounded domains.

The talk is based on results published in [1]-[3] and their development.

References

1. M. M.Malamud, To Birman–Krein–Vishik Theory, Doklady Mathematics, Vol. 107, No. 1 (2023), pp. 44–48.

2. M.M. Malamud, On the Birman problem on positive symmetric operators with compact inverse, Func. Anal. Appl., V.57, No 2 (2023), p. 111–116.

3. M. Malamud, Explicit Solution to the Birman Problem for the $2D$-Laplace operator, Russian Jour. of Math. Phys., 2024, Vol. 31, No.3, p. 495-504.

MIXED PERTURBATION MODELS OF INDEFINITE LAPLACIANS

Andrea Mantile

Laboratoire de Mathématiques de Reims, France

Sign-indefinite Laplacians appear in scalar models of composite EM-materials with piecewise-constants sign-changing coefficients. Such configurations have relevant applications, including plasmonic resonances and metamaterial design.

The problem of selfadjointness for indefinite Laplacians in bounded domains was adressed in [3]. The lack of coercivity of such operators entails a well-posedness issue for the scattering-resonance problem. This was first addressed in [1], [2] using the T-coercivity approach. An asymptotic characterization of the high-frequency scattering resonances has been provided in [4].

I reconsider such models in wiev of the mixed perturbation theory elaborated in [5]. Building on recent results obtained for acoustic operators in [6], I will provide the Q-function and justify a Kreĭn-type resolvent representation, providing the setting for a direct approach to resonances.

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SOLITON GAS FOR THE NONLINEAR SCHRÖDINGER EQUATION

Alexander Minakov

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We consider soliton gas for the focusing nonlinear Schrödinger equation, and study its long-time asymptotic properties. We show that the $x, t > 0$ half-plane is divided into several sharply separated regions, where the asymptotics is described in terms of hyperelliptic functions of genus from one to three. This is a joint work with Tamara Grava and Giuseppe Orsatti.

OPTIMAL EIGENVALUES ON A METRIC GRAPH WITH DENSITIES

Noema Nicolussi

University of Innsbruck, Austria

Motivated by the notion of conformal eigenvalues for surfaces, we investigate Laplacians on a metric graph with varying mass density. This setting provides a common framework for several well-studied classes of operators: discrete Laplacians, Dirichlet-to-Neumann operators on graphs, and Kirchhoff Laplacians (a.k.a. quantum graph operators).

Our main interest is the spectral optimization problem for eigenvalues with respect to the underlying mass density, which turns out to behave rather differently than in the manifold setting. In particular, we discuss the relation of optimal eigenvalues with geometrical properties, including a complete geometric description of the first optimal eigenvalue and a Weyl law.

Based on joint work with Kiyan Naderi (University of Innsbruck).

QUANTUM ZENO EFFECT

Saverio Pascazio

University of Bari, Italy

We review the history of the quantum Zeno effect, highlighting Pavel Exner's contributions to this topic.

RESONANCES IN QUANTUM GRAPHS

Jan Pekař

Charles University, Prague, Czech Republic &
Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czech Republic

In the first part we discuss earlier work by Exner and Lipovský [1], where they consider quantum graphs consisting of a compact part and semi-infinite leads. Such a system may contain embedded eigenvalues in the continuous spectrum, which, under perturbation, move into the second sheet of the complex energy surface and produce resonances. We also show how the scattering and resolvent resonances in quantum graphs coincide and how ”nothing is lost at the perturbation” in the sense of the number of poles. In the second part we then introduce a cut-off technique known since the eighties [2] to our quantum graph framework. Using it, one can identify resonances through the eigenvalue behavior of the system ”closed in a box.” We prove its validity, which was before done only in case of one-dimensional potential scattering [3], and illustrate it with examples.

References:

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WHO NEEDS FUNCTION SPACES?

Luboš Pick

Charles University in Prague, Czech Republic

I will survey some techniques which I came across during my research in the field of function spaces during the last three decades, and discuss their effectivity. I will point out some of the challenges that motivated the origin of new methods and of new function spaces. The talk is aimed for general mathematical audience. In particular, it is not tailored for experts in function spaces.

THE MAKAI INEQUALITY IN HIGHER DIMENSIONS

Rossano Sannipoli

Czech Technical University in Prague, Czech Republic

We discuss a sharp inequality for the torsional rigidity of an open, bounded and convex set in $\mathbb{R}^n$, in terms of the perimeter and the measure of the set. The result extends to arbitrary dimensions an inequality proved by E. Makai (1962) in the planar case, confirming a conjecture posed by L. Briani, G. Buttazzo and F. Prinari (2020). We also present quantitative estimates that provide further insight into the geometric structure and the thickness of optimizing sequences.

FROM SPECTRAL PROJECTIONS TO RESOLVENT BOUNDS

Petr Siegl

Graz University of Technology, Austria

Anharmonic oscillators with complex polynomial potentials $L = -\partial_x^2 + x^{2a} + ix^b$ in $L^2(\mathbb{R})$ with $a, b \in \mathbb{N}$ have rich spectral properties and were studied in various contexts. It is known that the spectral projections P_n of L have a Riesz property if $b < a - 1$ and, on the other hand, the resolvent norm of L grows super-polynomially in a large part of the right complex half-plane if $b > a - 1$.

We show that if $a - 1 < b < 2a$, then spectral projections P_n do not have a basis property, moreover, for $\sigma = [b - (a - 1)]/(1 + a)$ and $\gamma > 0$ small enough,

$$\limsup_{n \to \infty} \frac{\|P_n\|}{\exp(\gamma n^\sigma)} = \infty.$$

Proofs are based on two groups of results of an independent interest:

(a) the relationship between the behavior (growth) of the norms of projections $\|P_n\|$ and of the resolvent $\|(z - L)^{-1}\|$ outside of the spectrum $\sigma(L)$;

(b) the partial fraction decompositions of special meromorphic functions $1/F$, where $F(w) = \prod_{k=1}^{\infty} \left(1 + \frac{w}{a_k}\right)$, $a_{k+1} \geq a_k > 0$, $k \in \mathbb{N}$, and the generalization of the first resolvent identity.

The talk is based on a joint work with B. Mityagin (OSU, USA).

Spectral statistics of Markovian matrices arising in the spectral theory of graphs

Uzy Smilansky

Weizmann Institute of Science, Rechovot, Israel

The Markovian matrices and their origin in spectral graph theory will be introduced, and their essential properties and nature will be presented and explained. Such matrices can be associated with any given Hermitian matrix of finite dimension. Here, these Markovian matrices will be constructed for matrices picked from several random matrix ensembles, and their spectral densities will be derived and discussed.

Twisting in soft quantum waveguides

David Spitzkopf

Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czech Republic

We consider a Schrödinger operator on $\mathbb{R}^3$ with a potential obtained by rotating a non-radially symmetric, decaying profile along a longitudinal axis. Under the assumption of a non-trivial, compactly supported twist and an isolated ground state of the transverse operator, we prove a Hardy-type inequality at the spectral threshold with a two-parameter weight $(1 + s^2)^{-1}(1 + |t|^2)^{-1}$. This establishes subcriticality of the operator and extends the repulsive effect of twisting, known for hard waveguides, to the soft setting.

The spectral pair for half-line Schrödinger operators with complex integrable potentials

František Štampach

Czech Technical University in Prague, Czech Republic

We introduce the concept of a spectral pair for a half-line Schrödinger operator with a complex potential, which serves as a substitute for the spectral measure in a non-self-adjoint setting. Assuming integrable potentials, we prove existence of Jost solutions of a system of two equations naturally associated with the non-self-adjoint Schrödinger operator and present explicit formulas for the spectral pair in terms of these Jost solutions. The talk is based on a joint work with A. Pushnitski.

From AHOs to Painlevé

Miloš Tater

Nuclear Physics Institute of the Czech Academy of Sciences, Řež, Czech Republic

We discuss the asymptotic of the solvable part of the spectrum for the quasi-exactly solvable quartic oscillator. Further, we formulate a conjecture on the coincidence of the asymptotic shape of the level crossings of the latter oscillator with the asymptotic shape of zeros of the Yablonskii–Vorob'ev polynomials. This surprising connection relating the second Painlevé transcendent and anharmonic oscillators is studied using the isomondromic representation of the Painlevé II and the WKB method. Further, we review analogous relation between sextic anharmonic oscillator and Painlevé IV.

Continuum limit for discrete block operator matrices

Matěj Tušek

Czech Technical University in Prague, Czech Republic

Building upon a recent result by H. Cornean, H. Garde, and A. Jensen concerning continuum limits of discrete Dirac operators, we extend our analysis to a wide class of block operator matrices. This class includes the bilayer graphene Hamiltonian, among others. Our main goal is to find norm estimates for the difference between the resolvents of continuous operators and their discrete counterparts embedded in the continuum in a specific way. Consequently, we investigate whether the discretized operators converge to the original continuous operators as the mesh parameter tends to zero. While some discretization schemes lead immediately to the convergence in the generalized norm resolvent sense, others may require the addition of a suitable correction term to ensure the convergence.

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